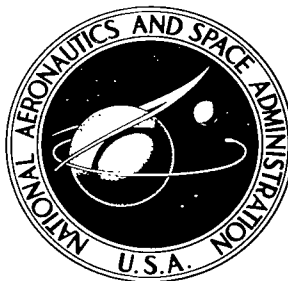


NASA TECHNICAL NOTE



NASA TN D-3773

C.1

NASA TN D-3773

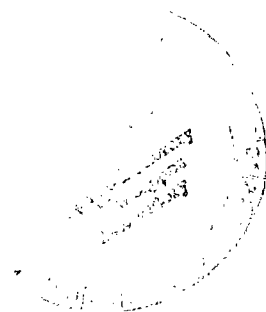


SIMULATION OF LIQUID-HYDROGEN TURBINE-TYPE FLOWMETER CALIBRATIONS USING HIGH-PRESSURE GAS

by Andrew J. Szaniszlo and Lloyd N. Krause

Lewis Research Center

Cleveland, Ohio



NASA TN D-3773

TECH LIBRARY KAFB, NM



0130420

**SIMULATION OF LIQUID-HYDROGEN TURBINE-TYPE FLOWMETER
CALIBRATIONS USING HIGH-PRESSURE GAS**

By Andrew J. Szaniszlo and Lloyd N. Krause

**Lewis Research Center
Cleveland, Ohio**

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 - Price \$1.00

SIMULATION OF LIQUID-HYDROGEN TURBINE-TYPE FLOWMETER

CALIBRATIONS USING HIGH-PRESSURE GAS

by Andrew J. Szaniszlo and Lloyd N. Krause

Lewis Research Center

SUMMARY

The accuracy of simulating liquid-hydrogen turbine-meter calibrations by utilizing nitrogen gas at liquid-hydrogen density is established for three different internal configurations of 1.5-inch turbine-type flowmeters. The average of the absolute percent differences for the three sets of turbine meters employed indicates that the liquid-hydrogen calibration factor at full scale can be simulated with 60-atmosphere nitrogen gas to 0.4 percent. For the majority of meters tested, liquid-hydrogen calibrations are simulated more closely by ambient-temperature, high-pressure nitrogen-gas calibrations than by water calibrations.

Turbine-meter repeatability, also, is determined for high-pressure nitrogen gas. Typical meter repeatability obtained with nitrogen gas for a 95-percent confidence band is ± 0.3 percent.

INTRODUCTION

For commonly used fluids, turbine-type flowmeters are extensively employed as metering devices because they are convenient to use and they have a linear digital-signal output. Because of these features, the turbine meter is also being used for cryogenic fluid metering. The increasing use of cryogenic fluids in research facilities and aerospace propulsion systems has required more accurate knowledge of turbine-type flowmeter cryogenic flow characteristics (refs. 1 and 2). Cryogenic calibrations of turbine-type flowmeters are time consuming and costly since few cryogenic facilities exist. It is advantageous, therefore, to determine how accurately turbine-meter calibrations with the less hazardous and less expensive fluids simulate liquid-hydrogen calibrations. Comparisons have been made between water and liquid-hydrogen calibrations. Results of these comparisons indicate that for inaccuracies less than 1 percent at full-scale flow, water calibrations are inadequate in predicting the meter constant for liquid-hydrogen

flow in turbine-type flowmeters (refs. 1, 3, and 4).

The primary purpose of this work was to determine experimentally the accuracy of simulating liquid-hydrogen calibrations using nitrogen gas. The nitrogen gas was at approximately ambient temperature and at a density equaling that of liquid hydrogen. Secondary objectives were to determine nitrogen-gas turbine-meter repeatability and to note any change in meter calibration due to installation and meter condition. Derivation of a theoretical temperature conversion factor that is applied to liquid-hydrogen calibration data for comparison with the nitrogen-gas calibrations is contained in appendix B.

The turbine-meter calibrations on liquid hydrogen and water used for comparison purposes in this report are discussed in detail in a companion report (ref. 4).

APPARATUS AND TEST PROCEDURE

A schematic drawing of the test facility is shown in figure 1. Nitrogen gas supplied from a portable gas trailer was throttled from 163 atmospheres to the desired operating pressure. The prime operating pressure level was 59 atmospheres with a maximum flow rate of 1.4 pounds per second, which permitted three to four turbine-meter calibrations to be obtained from one trailer. A sharp-edged orifice (modification of ref. 5) was used as the flow standard with a maximum pressure difference of 7 psi. The orifice, 50-diameter-length approach pipe, and the 25-diameter-length downstream pipe were calibrated as a unit for the determination of the discharge coefficient over the required Reynolds number range with water. Accuracy of the water stand used for this calibration was better than ± 0.3 percent. Differential pressure across the measuring orifice was determined with a null balance differential pressure transmitter. Measurement of the change in orifice differential pressure transmitter nonlinearity with the pressure level was accomplished with a mercury manometer for pressure levels to 100 atmospheres. Downstream of the orifice, the two horizontal turbine-meter test sections had an inter-

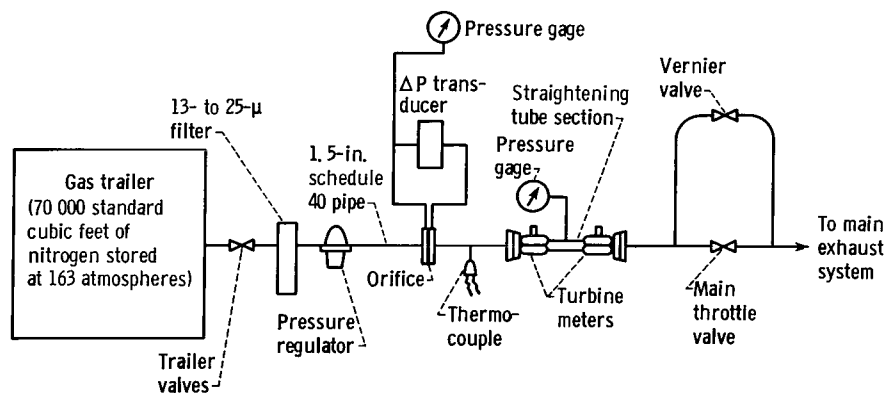


Figure 1. - Nitrogen gas flow facility.

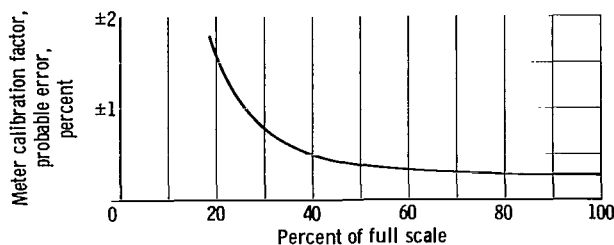


Figure 2. - Probable error of nitrogen-gas calibration factor against percent of nominal full-scale flow pulse rate.

posed straightening section of 19 tubes, each 15 tube diameters long. Operating pressure levels were measured to ± 1.5 psi at the straightening section. Gas temperature was measured with a thermocouple probe 5.5 pipe diameters downstream of the orifice (ref. 6). The mass flow rate was controlled by throttling valves 45 pipe diameters downstream of the test section.

Recording potentiometers were used to measure the thermocouple electromotive force and the orifice differential pressure signal. The inaccuracies in the temperature and orifice differential pressure measurements were determined to be within $\pm 0.4^\circ \text{R}$ and ± 0.01 psi, respectively.

After instrument calibration and a slow pressurization of the facility, nitrogen-gas flow was initiated and gradually increased to the maximum recommended flow rate for the turbine-type flowmeters run. Not until the temperature of the facility had stabilized, approximately 5 minutes later, were turbine-meter data recorded. Each data point consisted of a 1-minute run at constant flow to obtain an average turbine-meter pulse rate.

The probable error of the turbine-meter constant was analytically determined from consideration of possible instrument errors and is shown by figure 2. At low flow rates, the prime error contribution is from the orifice differential-pressure measurement.

Commercially available turbine meters of nominal 1.5-inch size with upstream straightening vanes and full-complement ball bearings of stainless steel from three different manufacturers were utilized in this investigation. Additional specifications of these meters are listed under Group I in reference 4.

RESULTS AND DISCUSSION

In this investigation, as previously mentioned, data were obtained for commercially available, 1.5-inch turbine meters by using nitrogen gas at a pressure simulating liquid-hydrogen density. Also from these tests, turbine-meter calibration repeatability was determined. Comparison of the 59-atmosphere nitrogen-gas calibrations to the temperature-corrected liquid-hydrogen calibrations, obtained from the facility of refer-

TABLE I. - TOTAL ACCUMULATED
RUNNING TIME ON TURBINE-TYPE
FLOWMETERS FOR NITROGEN
GAS, LIQUID HYDROGEN,
AND WATER

Meter	Total accumulated running time, hr		
	Water	Liquid hydrogen	Nitrogen
A1	7	12	5
A4	5	12	2
A5	7	16	7
B1	5	16	16
B2	5	21	3
B3	7	14	12
B4	5	12	13
B5	10	22	15
C1	5	14	5
C2	5	8	3
C4	5	20	5

ence 3, determined the accuracy of simulating liquid-hydrogen calibrations with nitrogen gas. Further comparisons are made with water calibrations. The total accumulated running time for the meters used in this report is shown in table I. For a given meter, total times for all fluids vary from 16 to 47 hours. The meter designation, for consistency, is the same as used in reference 4.

Simulation Considerations

The simulation of liquid-hydrogen flow through a given turbine flowmeter by means of another fluid requires that certain properties of the fluid be the same. Two of these properties are the following:

(1) The kinematic viscosity of the fluids should be the same to ensure that the Reynolds numbers of the flow through the turbine meter will be equal for a given flow velocity. This condition presents similar velocity profiles through the meter

(2) The density of the fluids should be the same to ensure that the torque developed on the turbine meter overcomes the retarding forces (bearing friction and magnetic drag) in the meter and that these forces be the same for both fluids at a given fluid velocity.

If liquid-hydrogen flow is simulated by room-temperature nitrogen at a pressure of about 60 atmospheres, the previous two conditions are approximately satisfied as shown in figure 3. Density-change effects on nitrogen-gas

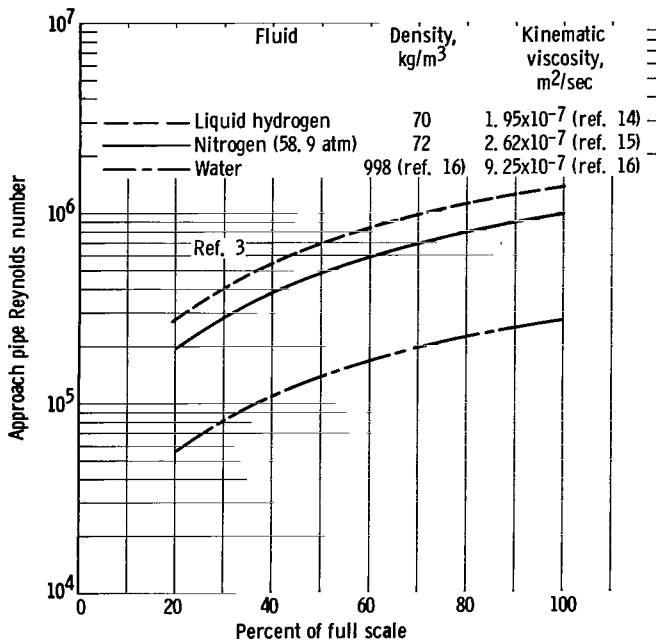


Figure 3. - Representative approach pipe Reynolds number for 1.5-inch meters against percent of full scale.

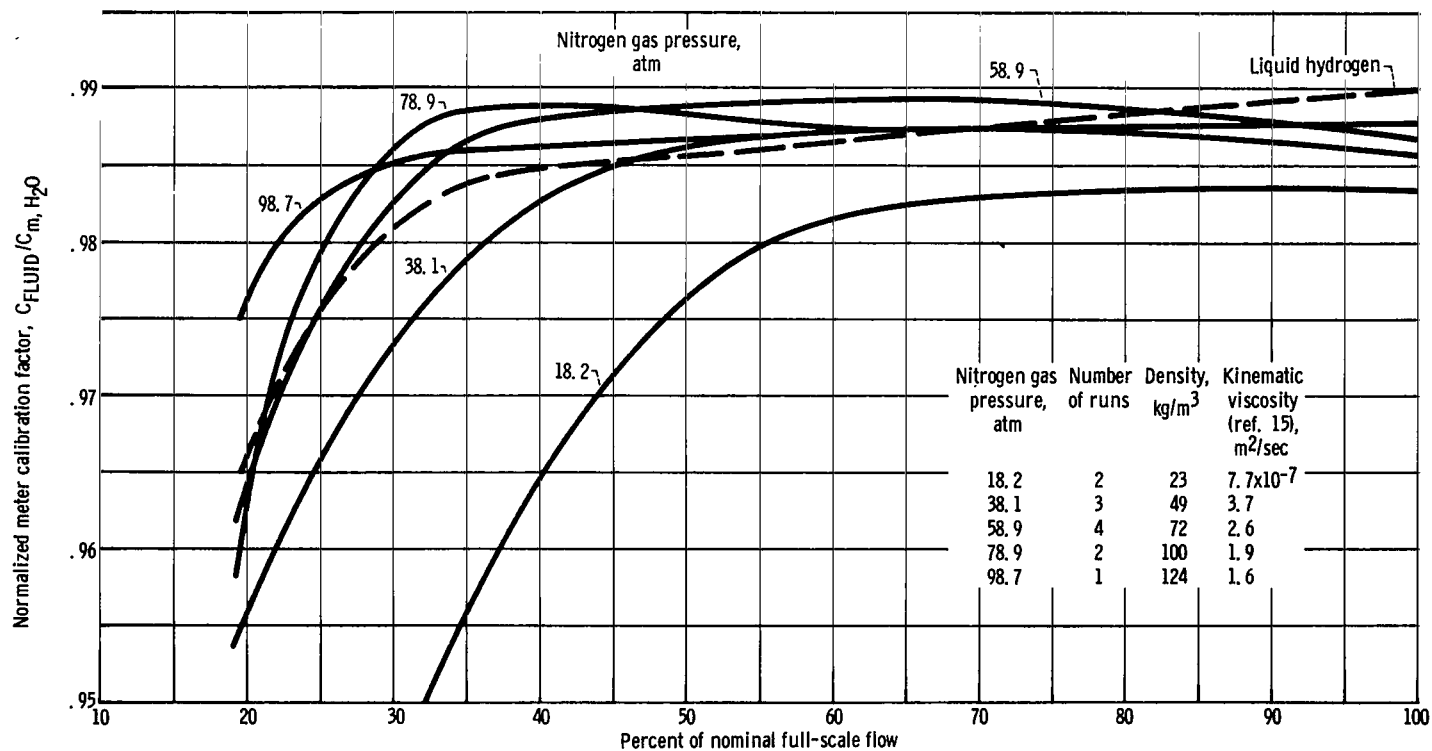


Figure 4. - Average curves of the density effect on the normalized meter calibration factor for meter B5.

calibrations for a single meter are illustrated in figure 4. Included for comparison is the temperature-corrected liquid-hydrogen calibration of the meter. The ordinate of figure 4 is presented in a nondimensional form by dividing the nitrogen-gas calibration factor C_{FLUID} (pulses/unit volume) by the calibration factor for water C_{m, H_2O} at full-scale flow, and the abscissa is expressed in terms of percent of the manufacturer's nominal full-scale pulse rate. The number used for C_{m, H_2O} is an average of the water calibrations obtained during this investigation. Wherever possible, all ensuing graphs are similarly presented. Figure 4 also shows that for a pressure at or greater than 38 atmospheres, the values of the average calibration factor for one meter at 50 percent of nominal full-scale flow or greater agree to within ± 0.2 percent with larger differences occurring at lower flow rates. Furthermore, the results for a gas pressure simulating liquid-hydrogen density and for a gas pressure simulating liquid-hydrogen kinematic viscosity (79 atm) are quite similar. For convenience, all subsequent nitrogen-gas data reported herein were taken at the density simulating pressure of 59 atmospheres.

A more complete simulation of liquid-hydrogen flow by another fluid would require that other conditions such as the following be satisfied:

(1) The fluid should have the same temperature as the liquid hydrogen to ensure that the dimensional changes due to temperature be the same.

(2) Bearing surface conditions should be the same in both fluids.

For the work reported herein, these last factors were not simulated.

Repeatability

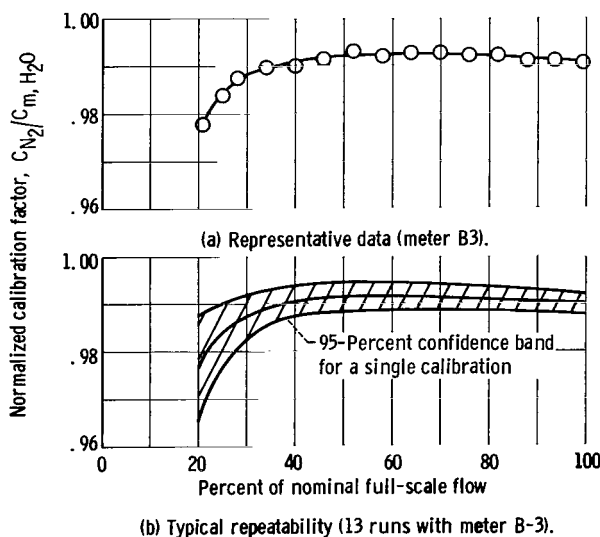


Figure 5. - Turbine meter calibrated in 58.9-atmosphere nitrogen gas.

A better interpretation of the results obtained from comparing the nitrogen-gas and liquid-hydrogen data is gained not only from using averaged calibration curves but also from knowing the respective calibration repeatability values. A typical set of data points for a single nitrogen-gas calibration is shown in figure 5(a) for one meter. The curve contains points for conditions of decreasing and increasing flow rate between set points. For any single calibration, the data points fall on a smooth curve. However, this curve will shift from calibration to calibration as shown by figure 5(b), which

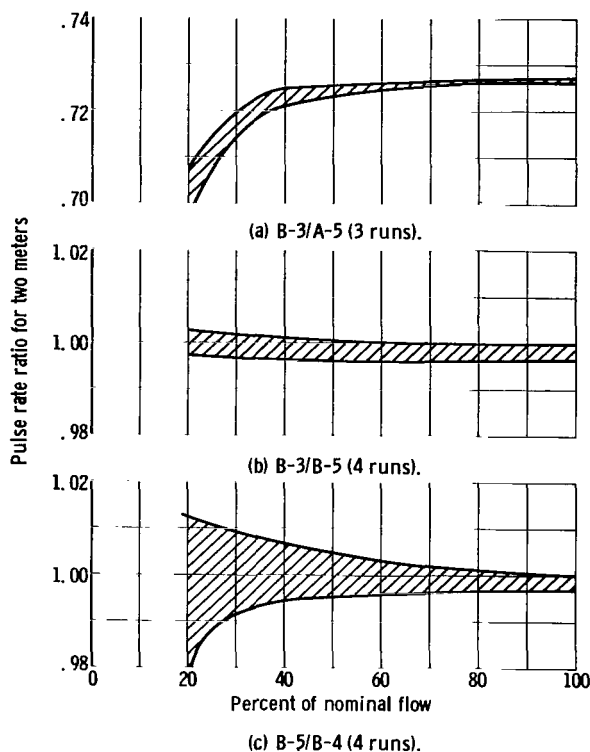


Figure 6. - Upstream meter to downstream meter, pulses-per-second ratio against percent of nitrogen gas full-scale flow of downstream meter.

represents 13 calibrations of the same meter as presented in figure 5(a). At a pulse rate greater than 40-percent full scale, the calibration repeatability is ± 0.3 percent as shown by figure 5(b) for a 95-percent confidence band of a finite number of runs (ref. 7).

In order to distinguish meter repeatability from the test-facility repeatability, several combinations of two meters were calibrated in series. The ratio of the pulse rate for the two meters run in series was computed and is shown by figure 6. For each pair of meters, several runs were made at random times during the test program, which generated the band shown for each meter combination. All measured variables are excluded except the counter data; consequently, figure 6 reveals that turbine-meter calibration factors can unpredictably shift with time. The three bands of figure 6 represent the average and the two extremes of repeatability of the meters tested.

The reason for the shifts in the meter calibrations is not known, but several factors can be listed that might contribute to the shifts:

- (1) Fine particles swept out of the nitrogen-gas supply bottle that passed the sintered-metal filter and deposited on any of the sliding surfaces of the meter
- (2) Change in the type and/or degree of meter sliding-surface contamination when alternately run on liquid hydrogen, water, and nitrogen gas
- (3) Random change in the bearing characteristics due to normal wear with time

Test-Section Inlet

In order to determine the effect that the existing discontinuity in the geometry of the approach pipe has on the turbine-meter calibration factor, a removable constricting insert was installed. The geometry of the insert and of a typical turbine-meter inlet is shown in figure 7. The insert reduced the pipe area by 45 percent and had a length to internal diameter ratio of 4.9. Location of the insert exit depended on the meter being tested and ranged from 1.6 to 2.8 diameters upstream of the meter-blade location. The

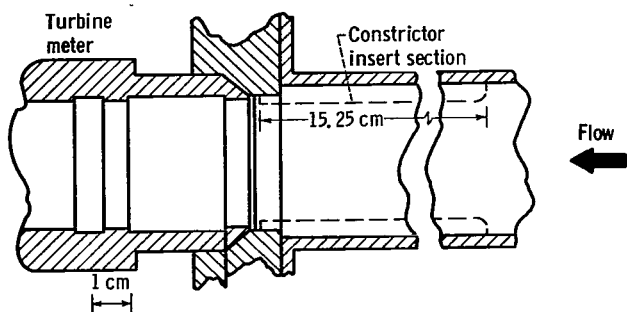


Figure 7. - Turbine-meter inlet configuration.

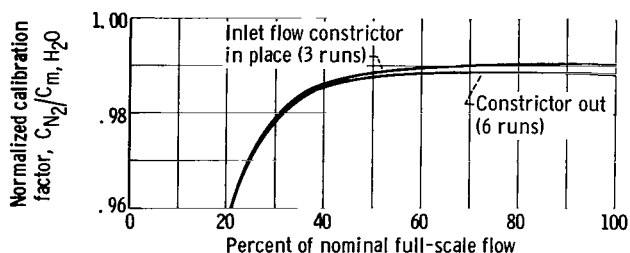


Figure 8. - Effect of inlet geometry change on turbine-meter nitrogen-gas calibration factor (meter B1).

average curves for one meter with and without the constricting insert in place are shown in figure 8. With the insert in place, the meter calibration factor increased 0.25 percent at the full-scale flow rate. A 45-percent area contraction produced only a 0.25-percent increase in the nominal full-scale calibration factor C_{nfs} . A possible explanation for this calibration-factor increase is that some of the fluid velocity increase due to the area contraction is sensed at the meter blades and thus causes an increase in rotor speed.

Meter Interaction

The APPARATUS AND TEST PROCEDURE section stated that the test section consisted of two turbine meters in series separated by a straightening-vane section. Table II shows the influence

TABLE II. - CALIBRATION-FACTOR SHIFTS DUE TO POSITION

CHANGE RELATIVE TO ANOTHER TURBINE METER

SIMULTANEOUSLY RUN IN SERIES

Meter	Total number of calibrations	$\left(\frac{C_{US} - C_{DS}}{C_{DS}} \right)_{N_2}$, percent		Other meter type run in series
		50-percent full scale	100-percent full scale	
A1	2	-0.20	-0.20	C
A5	2	-1.02	-.86	B
B1	12	0.40	0.39	B and C
B5	2	.26	.15	A and B
C1	4	-0.53	-0.61	A
C2	4	-.23	-.27	B
C4	2	-.20	-.20	A

of relative position on the calibration factor for each meter. The table indicates a greater meter factor for downstream meters A and C, while meter B with constant blade angle opposes the trend. Because of the change in calibration factor with position, only upstream meter data will be employed for determining the accuracy of simulating liquid-hydrogen flow with nitrogen gas.

Also, no discernable effect was determined on the calibration factor by tightening torques to 250 pound-feet on the flare fittings of these meters.

Liquid-Hydrogen Calibration Simulation

Comparison of liquid-hydrogen calibrations to nitrogen-gas calibrations for a representative sample of nine turbine meters is shown in figure 9. For additional comparison, water calibrations are included in the figure. The water and liquid-hydrogen data presented are part of the same data used in reference 4. In order to compare the three fluids on an equal temperature basis, the hydrogen data has been adjusted to room temperature by using the multiplication factor derived in appendix B. The curves shown represent the average of multiple calibrations except for the three cases of single nitrogen runs.

An examination of figure 9 based on the different meter sets reveals the following:

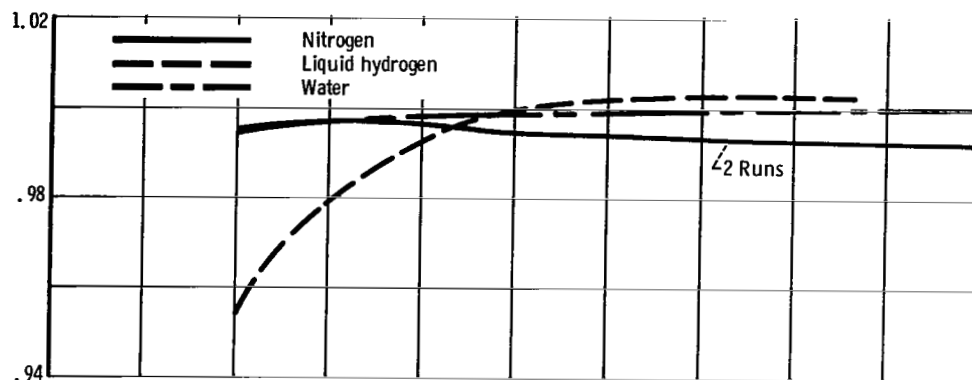
(1) Results of only two meters of set A are presented with a limited number of runs; however, the shape of the curves for set A follow the water rather than liquid hydrogen.

(2) For the set B meters, 60-atmosphere nitrogen gas definitely does a better job than water in simulating the liquid-hydrogen calibrations. The curves for nitrogen of every set B meter presented are closer to the liquid-hydrogen curves in both shape and level than to those of water.

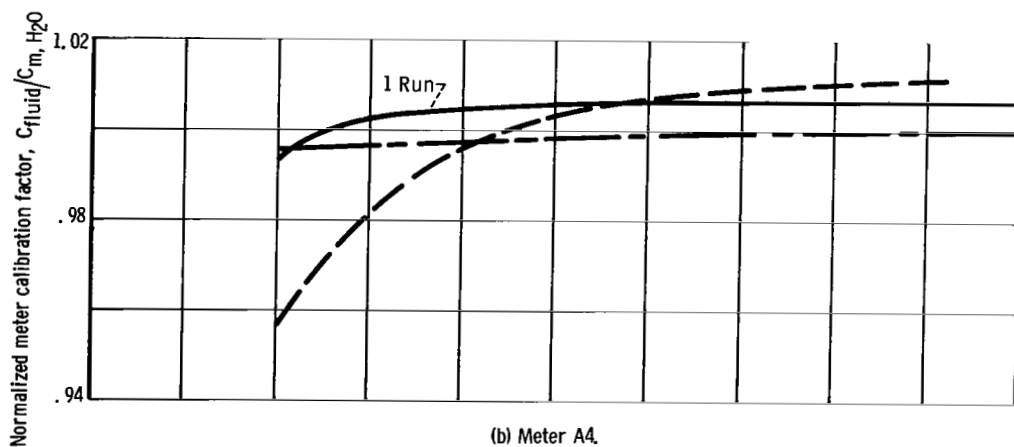
(3) Two of the three meters of set C had similar curves for all three fluids. The results of meter C4 shown in figure 9 were included to show the nitrogen calibration for a case where the liquid-hydrogen and water calibrations deviated greatly. At the high flows, the nitrogen calibration follows liquid hydrogen rather than water. Normally, a meter with characteristics such as those of C4 would be rejected for liquid-hydrogen service based on the acceptance standards given in reference 4.

The calibration-simulation results compiled in table III show that the average of the absolute percent differences for liquid-hydrogen calibration simulation with water is approximately 0.5 percent at the nominal full-scale value for the three sets of meters run.

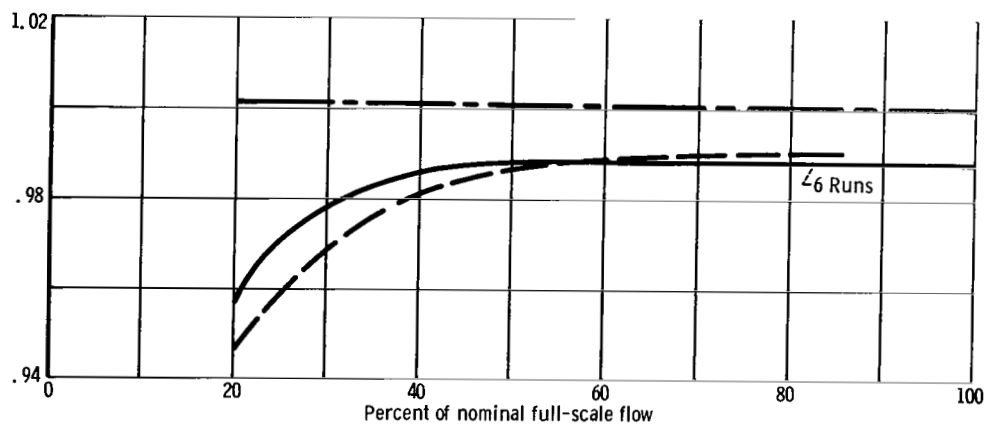
Substitution of 60-atmosphere nitrogen gas for water not only improves liquid-hydrogen calibration-simulation accuracy at the full-scale point, but also at the lower flow rates. For the three different sets of turbine meters run, table III shows that cali-



(a) Meter A1.

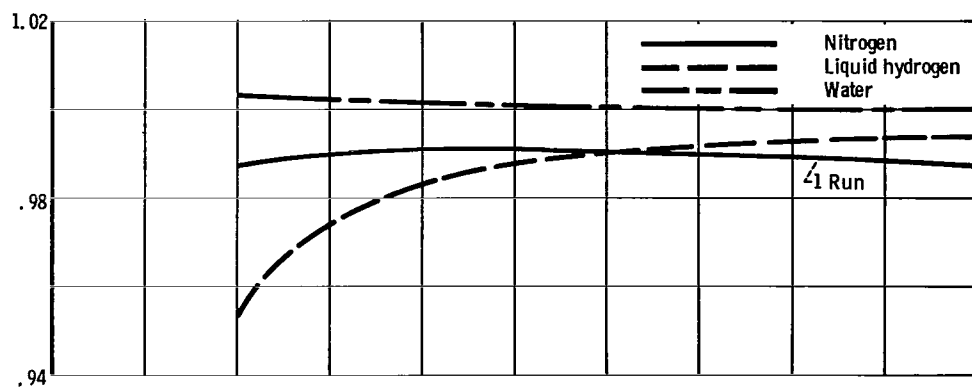


(b) Meter A4.

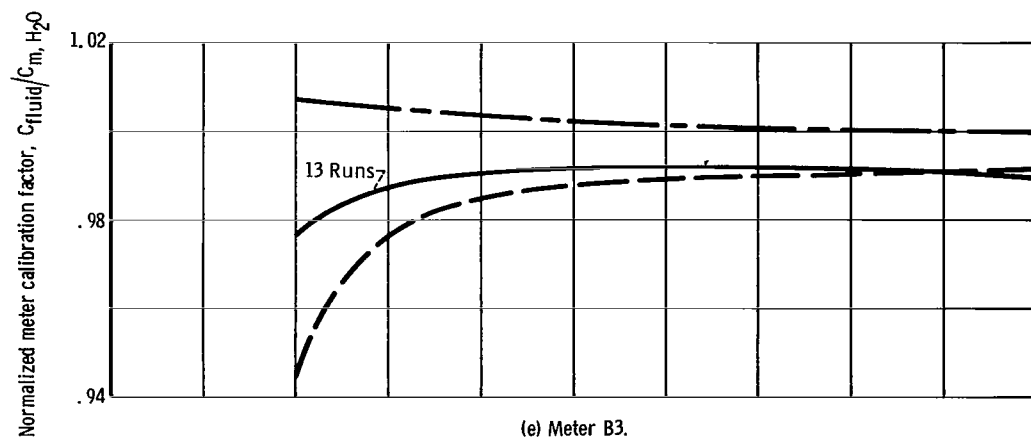


(c) Meter B1.

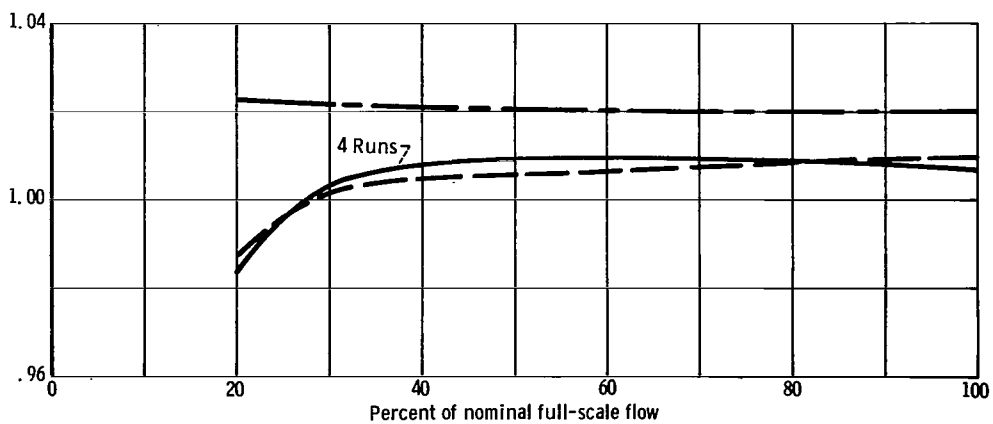
Figure 9. - Comparison of turbine-meter calibrations for water, liquid hydrogen, and nitrogen.



(d) Meter B2.

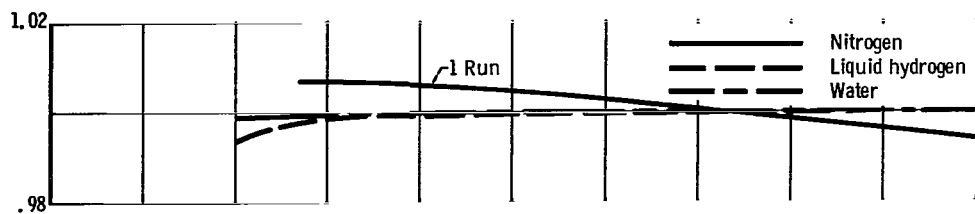


(e) Meter B3.

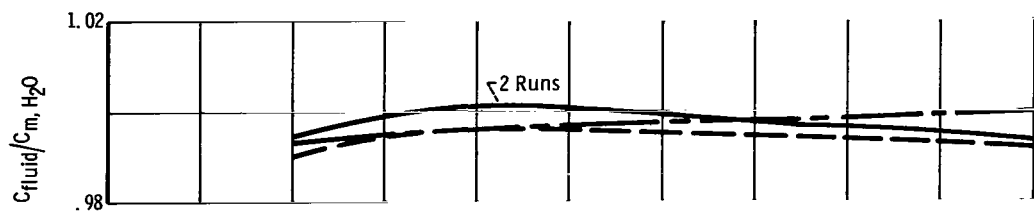


(f) Meter B5.

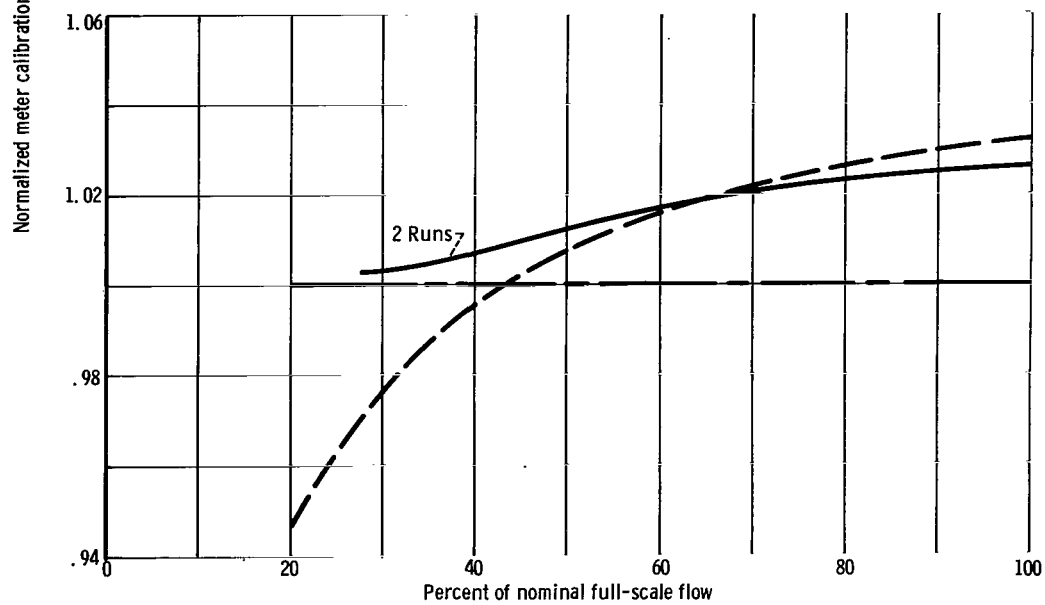
Figure 9. - Continued.



(g) Meter C1.



(h) Meter C2.



(i) Meter C4.

Figure 9. - Concluded.

TABLE III. - SUMMARY OF COMPARISON OF
TURBINE-METER CALIBRATIONS

Meter set	Number of meters run with nitrogen gas	Average difference in percent between liquid hydrogen calibrations and those of water and nitrogen for stated percent of maximum flowrate			
		Nitrogen gas		^a Water	
		40	100	40	100
A	2	-0.6	0.7	-0.4	0.7
B	4	-.6	.3	-1.6	-.7
C	3	-.7	.3	-.6	-.2

^aData from ref. 4 after adjusting the liquid-hydrogen calibrations with the temperature factor obtained in appendix B. A greater number of meters per set are represented by the results than are indicated for nitrogen gas.

bration simulations can be had, averaging absolute values, to 0.4 percent at full-scale flow and to 0.6 percent at 40-percent full-scale flow. Further, averaging the three meter sets gives a simulation to within 0.6 percent from maximum flow to 40 percent of the maximum flow rate.

SUMMARY OF RESULTS

The accuracy of simulating liquid-hydrogen flow with 60-atmosphere nitrogen gas was determined by tests on three different internal configurations of 1.5-inch turbine-type flowmeters. Also determined were turbine-meter repeatability, test-section inlet effects, and turbine-meter interaction effects when two meters were operated in series. The noteworthy results of this study indicate the following:

1. Liquid-hydrogen calibrations can be simulated with 60-atmosphere nitrogen gas to 0.4 percent, average of absolute differences, at 100-percent full-scale flow and to 0.6 percent at 40-percent full-scale flow.
2. For the majority of meters tested, liquid-hydrogen calibrations are simulated more closely by ambient-temperature, high-pressure nitrogen-gas calibrations than by water calibrations.
3. Calibrations of the turbine-type flowmeters reported herein do unpredictably shift with time which results in a typical calibration repeatability for high-pressure nitrogen gas of ± 0.3 percent for a 95-percent confidence band.

4. Variation in installation torque on the meter body produces no change in the calibration factor.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 20, 1966,
128-31-06-77-22.

APPENDIX A

SYMBOLS

A_{BR}	transverse cross-sectional area of bladed rotor	Q_T	total volume flow rate through turbine meter
A_H	inside meter housing transverse cross-sectional area at blade section	R	some average value of blade radius from axis of symmetry
A_R	hub transverse cross-sectional area of bladed rotor	T	operating temperature of turbine meter
C_{DS}	calibration factor for meter located downstream, pulses/unit volume	T_O	calibration temperature of turbine meter
C_{FLUID}	calibration factor in fluid, pulses/unit volume	ΔT	$T - T_O$
C_{m, H_2O}	asymptotic value for water calibration factor	\bar{V}	averaged fluid velocity over blade length
C_{N_2}	nitrogen gas calibration factor	\bar{V}_N	average fluid velocity in an equivalent plain annulus
C_{nfs}	nominal full-scale calibration factor	\bar{V}_S	velocity of fluid through annular blade-tip clearance area
C_{US}	calibration factor for meter located upstream, pulses/unit volume	α	blade angle with respect to axis of symmetry
D_{BR}	bladed rotor tip-to-tip diameter	β_a	blade axial coefficient of thermal expansion
D_H	internal diameter of meter housing at blade section	β_H	meter housing coefficient of thermal expansion
D_R	hub diameter of bladed rotor	β_R	blade and rotor coefficient of thermal expansion
K	temperature correction factor	β_t	blade transverse coefficient of thermal expansion
Q_{LOSS}	volume flow rate through annular blade-tip clearance area	ξ	ratio of velocity at a blade-tip clearance distance from meter housing to average fluid velocity in an equivalent plan annulus
Q_{MT}	volume flow rate through cross-sectional area of bladed rotor		

ω bladed-rotor angular velocity

ω_{LH_2} bladed-rotor angular velocity in
liquid hydrogen

ω_{N_2} bladed-rotor angular velocity in
gaseous nitrogen

APPENDIX B

TURBINE-TYPE FLOWMETER, ANALYTICAL DERIVATION OF THERMAL-CORRECTION FACTOR FOR NONZERO BLADE CLEARANCE

A thermal-correction factor for nonzero blade clearance is derived for the case of identical blade and rotor hub material but for which the meter housing material can differ from that of the blade material.

For an unretarded spinning turbine blade, the following equation (ref. 8) holds:

$$\tan \alpha = \frac{R\omega}{\bar{V}} \quad (B1)$$

where α is the blade angle with respect to the axis of symmetry, ω the blade angular velocity, \bar{V} the fluid velocity averaged across the blade length, and R an appropriate value of blade radius from the axis of symmetry.

The metered volume flow rate Q_{MT} that crosses only the area swept out by the blades is the difference between the total volume flow rate through the turbine meter at the blade location and the volume flow rate through the annular blade-tip clearance area; that is,

$$Q_{MT} = \underbrace{(A_{BR} - A_R)\bar{V}}_{\substack{\text{Fluid through} \\ \text{blade area} \\ \text{annulus}}} = \underbrace{(A_H - A_R)\bar{V}_N}_{\substack{\text{Total fluid} \\ \text{through} \\ \text{turbine meter}}} - \underbrace{(A_H - A_{BR})\bar{V}_S}_{\substack{\text{Fluid through} \\ \text{tip clearance} \\ \text{annulus}}} = Q_T - Q_{LOSS} \quad (B2)$$

where A is a cross-sectional area swept out by a radius normal to the axis of symmetry, and the subscripts are S , annular blade-tip clearance area; R , rotor hub; BR , blade and rotor hub; and H , meter housing.

Solving equations (B1) and (B2) for ω yields

$$\omega = \left[\left(\frac{A_H - A_R}{A_{BR} - A_R} \right) \bar{V}_N - \left(\frac{A_H - A_{BR}}{A_{BR} - A_R} \right) \bar{V}_S \right] \frac{\tan \alpha}{R} \quad (B3)$$

and the total differential of ω is

$$d\omega = \left[\left(\frac{A_H - A_R}{A_{BR} - A_R} \right) \bar{V}_N - \left(\frac{A_H - A_{BR}}{A_{BR} - A_R} \right) \bar{V}_S \right] d\left(\frac{\tan \alpha}{R} \right) + \frac{\tan \alpha}{R} d \left[\left(\frac{A_H - A_R}{A_{BR} - A_R} \right) \bar{V}_N - \left(\frac{A_H - A_{BR}}{A_{BR} - A_R} \right) \bar{V}_S \right] \quad (B4)$$

Finding $(d\omega/\omega)$ by expanding the differential terms of equation (B4), dividing equation (B4) by (B3), and simplifying by the use of equation (B2) yield

$$\frac{d\omega}{\omega} = \frac{d(\tan \alpha)}{\tan \alpha} - \frac{dR}{R} + \frac{Q_T}{Q_{MT}} \left[\frac{d\left(\frac{A_H - A_R}{A_{BR} - A_R} \right)}{\frac{A_H - A_R}{A_{BR} - A_R}} + \frac{d\bar{V}_N}{\bar{V}_N} \right] - \frac{Q_{LOSS}}{Q_{MT}} \left[\frac{d\left(\frac{A_H - A_{BR}}{A_{BR} - A_R} \right)}{\frac{A_H - A_{BR}}{A_{BR} - A_R}} + \frac{d\bar{V}_S}{\bar{V}_S} \right] \quad (B5)$$

Noting that

$$\frac{d\bar{V}_N}{\bar{V}_N} = \frac{-d(A_H - A_R)}{A_H - A_R} \quad \text{from} \quad \frac{dQ_T}{Q_T} = 0$$

and setting $A_H = A_{BR}$, $Q_T = Q_{MT}$, and $Q_{LOSS} = 0$ yield

$$\frac{d\omega}{\omega} = \frac{d(\tan \alpha)}{\tan \alpha} - \frac{dR}{R} - \frac{d(A_H - A_R)}{A_H - A_R} \quad (B6)$$

which agrees with the equation derived in reference 9 for zero blade clearance.

If only finite variable changes are considered from now on, the first term of equation (B5) for nonisotropic turbine blade material is

$$\frac{\Delta(\tan \alpha)}{\tan \alpha} = \frac{(\beta_t - \beta_a) \Delta T}{1 + \beta_a \Delta T} \quad (B7)$$

where β_t is the blade transverse coefficient of thermal expansion, and β_a is the axial coefficient of thermal expansion. It is hereinafter assumed that $\beta_t = \beta_a$.

The second term of equation (B5) involves a blade radius and is

$$\frac{\Delta R}{R} = \beta \Delta T \quad (B8)$$

Using the binomial series yields the following approximation for the area ratios:

$$\frac{A_H - A_R}{A_{BR} - A_R} = \frac{D_H^2 - D_R^2 + 2 \Delta T (\beta_H D_H^2 - \beta_R D_R^2)}{(D_{BR}^2 - D_R^2) (1 + 2\beta_R \Delta T)} \quad (B9)$$

and

$$\frac{A_H - A_{BR}}{A_{BR} - A_R} = \frac{D_H^2 - D_{BR}^2 + 2 \Delta T (\beta_H D_H^2 - \beta_R D_{BR}^2)}{(D_{BR}^2 - D_R^2) (1 + 2\beta_R \Delta T)} \quad (B10)$$

If only the cases for which $2\beta \Delta T \ll 1$ are considered, the following relations are obtained from equations (B9) and (B10):

$$\frac{\Delta \left(\frac{A_H - A_R}{A_{BR} - A_R} \right)}{\left(\frac{A_H - A_R}{A_{BR} - A_R} \right)_{T=T_0}} = 2\beta_R \Delta T \frac{\left(\frac{\beta_H}{\beta_R} - 1 \right) D_H^2}{D_H^2 - D_R^2} \quad (B11)$$

and

$$\frac{\Delta \left(\frac{A_H - A_{BR}}{A_{BR} - A_R} \right)}{\left(\frac{A_H - A_{BR}}{A_{BR} - A_R} \right)_{T=T_0}} = 2\beta_R \Delta T \frac{\left(\frac{\beta_H}{\beta_R} - 1 \right) D_H^2}{D_H^2 - D_{BR}^2} \quad (B12)$$

It is again noted that

$$\frac{\Delta \bar{V}_N}{\bar{V}_N} = \frac{-\Delta(A_H - A_R)}{(A_H - A_R)_{T=T_0}} = -2\beta \Delta T \frac{\left(\frac{\beta_H}{\beta_R} D_H^2 - D_R^2 \right)}{D_H^2 - D_R^2} \quad (B13)$$

The ratios of volume flow rate in equation (B5) can be expressed as follows:

$$\frac{Q_T}{Q_{MT}} = \left(\frac{A_H - A_R}{A_{BR} - A_R} \right) \frac{\bar{V}_N}{\bar{V}} = 1 + \left(\frac{D_H^2 - D_{BR}^2}{D_{BR}^2 - D_R^2} \right) \frac{\bar{V}_S}{\bar{V}} \quad (B14)$$

and

$$\frac{Q_{LOSS}}{Q_{MT}} = \left(\frac{A_H - A_{BR}}{A_{BR} - A_R} \right) \frac{\bar{V}_S}{\bar{V}} = \left(\frac{D_H^2 - D_{BR}^2}{D_{BR}^2 - D_R^2} \right) \frac{\bar{V}_S}{\bar{V}} \quad (B15)$$

All meter dimensions are determined for the nitrogen-gas calibration temperature. Also, each value of the coefficient of thermal expansion is an average value determined by the size of the temperature difference between the operation temperature and the calibration temperature.

When equations (B8), (B11), (B12), (B13), (B14), and (B15) are substituted into equation (B5), the following result is obtained:

$$\frac{\Delta \omega}{\omega} = -\beta_R \Delta T \left\{ 3 + 2 \frac{\bar{V}_S}{\bar{V}} \left[\frac{\left(\frac{\beta_H}{\beta_R} D_H^2 - D_{BR}^2 \right)}{D_{BR}^2 - D_R^2} \right] \right\} - \frac{\bar{V}_S}{\bar{V}} \left(\frac{D_H^2 - D_{BR}^2}{D_{BR}^2 - D_R^2} \right) \frac{\Delta \bar{V}_S}{\bar{V}_S} \quad (B16)$$

The last term of equation (B16) can be ignored for turbine meters with a small blade-tip clearance and a small difference of Reynolds number between the operation fluid and calibration fluid flow. Furthermore, the velocity ratio in equation (B16) can be written as follows for incompressible flow:

$$\frac{\bar{V}_S}{\bar{V}} = \frac{D_{BR}^2 - D_R^2}{D_H^2 - D_R^2} \left(\frac{\bar{V}_N}{\bar{V}_S} - \frac{D_H^2 - D_{BR}^2}{D_H^2 - D_R^2} \right)^{-1} \quad (B17)$$

Hence, equation (B16) becomes

$$\frac{\Delta\omega}{\omega} = -\beta_R \Delta T \left[3 + 2 \frac{\left(\frac{\beta_H}{\beta_R} D_H^2 - D_{BR}^2 \right)}{D_H^2 - D_R^2} \left(\frac{\bar{V}_N}{\bar{V}_S} - \frac{D_H^2 - D_{BR}^2}{D_H^2 - D_R^2} \right)^{-1} \right] \quad (B18)$$

where $\Delta\omega/\omega$ is the relative ω change due to $T - T_O$ or $(\omega_{LH_2} - \omega_{N_2})/\omega_{N_2}$, T the turbine-meter operating temperature, and T_O the calibration temperature.

Equation (B18) assumes the following:

- (1) Rotor retarding torques do not exist.
- (2) Blade blockage is zero.
- (3) Blade material is isotropic.
- (4) Blade and rotor are of the same material.
- (5) Meter housing material can differ from that of the rotor.
- (6) There is a small difference of Reynolds number between the operation fluid and calibration fluid flow.
- (7) $2\beta \Delta T \ll 1$.

TABLE IV. - CALCULATED RESULTS
OF EQUATION (B18)

Meter set	ξ	$\frac{\Delta\omega}{\omega}$	$K = \left(1 + \frac{\Delta\omega}{\omega}\right)^{-1}$
A	0.89	6.81×10^{-3}	0.9932
B	.76	6.24	.9938
C	.60	5.96	.9941

For a plain annulus and a Reynolds number greater than 10^5 for nitrogen gas, the fluid velocity at the blade tip for each meter geometry is about ξ of the average velocity \bar{V}_N as shown in table IV (ref. 10). Because of possible transverse flow near the wall at the turbine-meter blade section, the velocity at the wall could be less than the velocity in an equivalent plain annulus section (ref. 11). As an approximation at the turbine-meter blade location, therefore, the average fluid velocity \bar{V}_S near the wall is assumed to be one-half of the velocity that

would exist in a plain annulus of equal area at a distance from the wall equal to the blade-tip clearance (ref. 12).

From reference 13, an average β_H is 6.1×10^{-6} inch per inch per $^{\circ}\text{F}$ for 303 stainless steel, and β_R is 3.9×10^{-6} inch per inch per $^{\circ}\text{F}$ for 17-4 PH stainless steel for a ΔT of -447°F .

The calculated results from equation (B18) are presented in table IV. An average value of 0.994 for the temperature multiplication correction factor K was applied to the liquid-hydrogen data in this report.

REFERENCES

1. Bucknell, Richard L.; Lowler, Thomas D.; and Street, Robert L.: Cryogenic Flow Measurement. Standards Laboratories and Measurement Standards. Vol. 19, Pt. I of the 19th Annual Conference, Instrument Society of America, 1964.
2. Alspach, W. J.; and Flynn, T. M.: Considerations When Using Turbine-Type Flowmeters in Cryogenic Service. Advances in Cryogenic Engineering. Vol. 10, Sections A-L. K. D. Timmerhaus, ed., Plenum Press, 1965, pp. 246-252.
3. Minkin, Herbert L.; and Hobart, Howard F.: Liquid-Hydrogen-Flowmeter Calibration Facility; Preliminary Calibrations on Some Head-Type and Turbine-Type Flowmeters. NASA TN D-577, 1961.
4. Minkin, Herbert L.; Hobart, Howard F.; and Warshawsky, I.: Performance of Turbine-Type Flowmeters in Liquid Hydrogen. NASA TN D-3770, 1966.
5. Flow Meter Committee of the Verein deutscher Ingenieure: Standards for Discharge Measurement with Standardized Nozzles and Orifices. German Industrial Standard 1952, Fourth ed. NACA TM-952, 1940.
6. Anon.: Flow Measurement. Chapter 4 of Power Test Codes Supplements - Instruments and Apparatus. PTC 19.4-1959, ASME, 1959.
7. Dixon, Wilfred J.; and Massey, Frank J., Jr.: Introduction to Statistical Analysis. Second ed., McGraw-Hill Book Co., Inc., 1957.
8. Lee, W. F. Z.; and Evans, H. J.: Density Effect and Reynolds Number Effect on Gas Turbine Flowmeters. J. Basic Eng., vol. 87, no. 4, Dec. 1965, pp. 1043-1057.
9. Grey, Jerry: Calibration of Turbine Flowmeters for Cryogenic Operation. ARS J., vol. 30, no. 2, Feb. 1960, pp. 192-193.
10. Knudsen, James G.; and Katz, Donald L.: Fluid Dynamics and Heat Transfer. McGraw-Hill Book Co., Inc., 1958.
11. Herzig, Howard Z.; Hansen, Arthur G.; and Costello, George R.: A Visualization Study of Secondary Flows in Cascades. NACA TR-1163, 1954.
12. Markov, N. M.: Calculation of the Aerodynamic Characteristics of Turbine Blading. Associated Technical Services, Inc., 1958.
13. Schwartzberg, F. R.; Osgood, S. H.; Keys, R. D.; and Kiefer, T. F.: Cryogenic Materials Data Handbook. (Rep. No. AFML-TDR-64-280, DDC No. AD-609562), Martin Co., Aug. 1964.

14. Anon.: Cryogenic Engineering Data Manual. Manual WRE 95, Dept. of Supply, Australian Defense Scientific Service, Weapons Research Establishment, Salisbury, South Australia, Oct. 1961.
15. Hilsenrath, Joseph, et al.: Tables of Thermodynamic and Transport Properties of Air, Argon, Carbon Dioxide, Carbon Monoxide, Hydrogen, Nitrogen, Oxygen, and Steam. Pergamon Press, 1960.
16. Vennard, John K.: Elementary Fluid Mechanics. Third ed., John Wiley & Sons, Inc., 1954.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546